

# Chapter 3

1

## *INTENSITY TRANSFORMATIONS AND SPATIAL FILTERING*

# Agenda

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- This lecture will cover:
- **What Is Image Enhancement?**
- **Intensity Transformation Functions**
  - Piecewise-Linear Transformation Functions
- **Histogram Processing**
  - Histogram Equalization.
  - Histogram Matching (Specification).
  - Local Histogram Processing.
  - Using Histogram Statistics for Image Enhancement.
- **Fundamentals of Spatial Filtering**
  - The Mechanics of Spatial Filtering.
  - Spatial Correlation and Convolution.
  - Vector Representation of Linear Filtering.
- **Smoothing Spatial Filters**
  - Smoothing Linear Filters.
  - Order-Statistic (Nonlinear) Filters.

# AGENDA (Cont...)

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- **Sharpening Spatial Filters**
  - Using the Second Derivative for Image Sharpening.
  - Unsharp Masking and Highboost Filtering.
  - Using First-Order Derivatives for (Nonlinear) Image Sharpening.
- **Combining Spatial Enhancement Methods**

# What Is Image Enhancement?

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- Image enhancement is the process of making images more useful.
- The reasons for doing this include:
  - Highlighting interesting detail in images.
  - Removing noise from images.
  - Making images more visually appealing.

# Piecewise-Linear Transformation Functions

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- **Contrast stretching**
- **Intensity-level slicing**
- **Bit-plane slicing**

# Piecewise-Linear Transformation Functions (Cont...)

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## • Contrast stretching

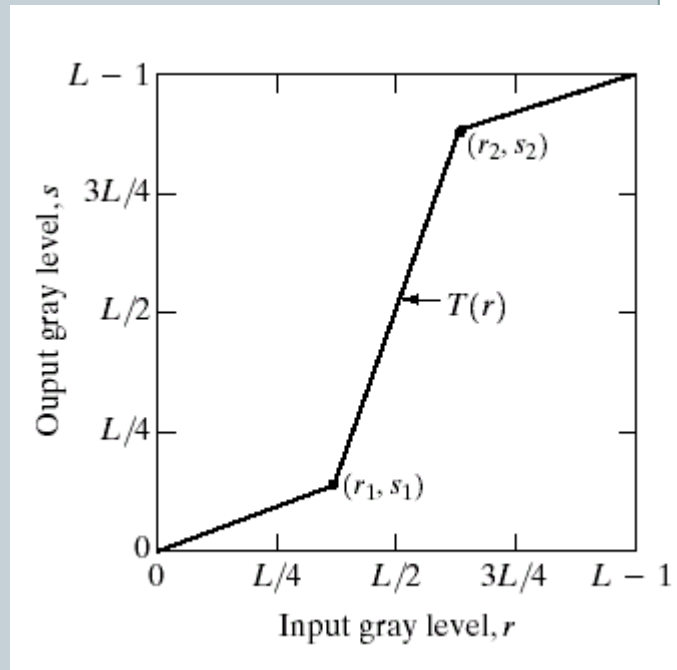
One of the simplest piecewise linear functions. Low-contrast images can result from poor illumination, lack of dynamic range in the imaging sensor or even wrong setting of lens aperture during image acquisition.

**Idea:** increase the dynamic range of the gray levels in the image being processed.

The locations of points  $(r_1, s_1)$  and  $(r_2, s_2)$  control the shape of the transformation function.

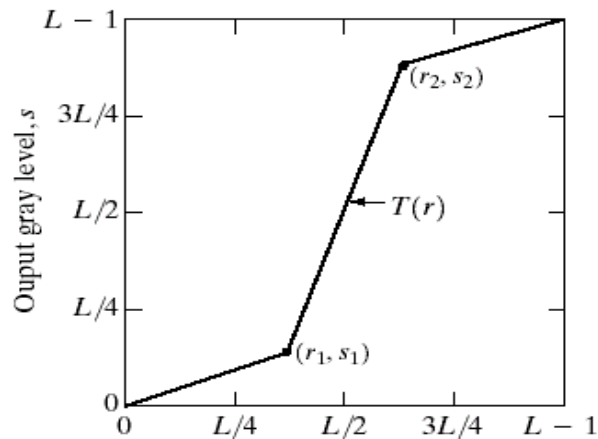
If  $r_1 = s_1$  and  $r_2 = s_2$ : The transformation is a linear function that produces no changes in gray levels.

If  $r_1 = r_2$ ,  $s_1=0$  and  $s_2=L-1$ : Transformation becomes a thresholding function that create a binary image.



# Piecewise-Linear Transformation Functions (Cont...)

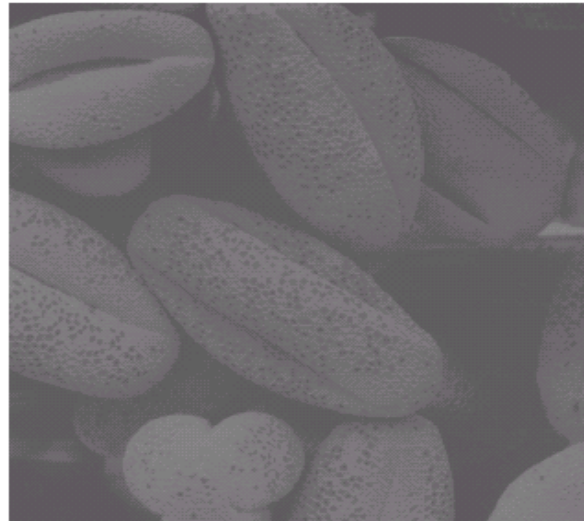
7



$$(r_2, s_2) = (r_{\max}, L - 1)$$

Input gray level,  $r$

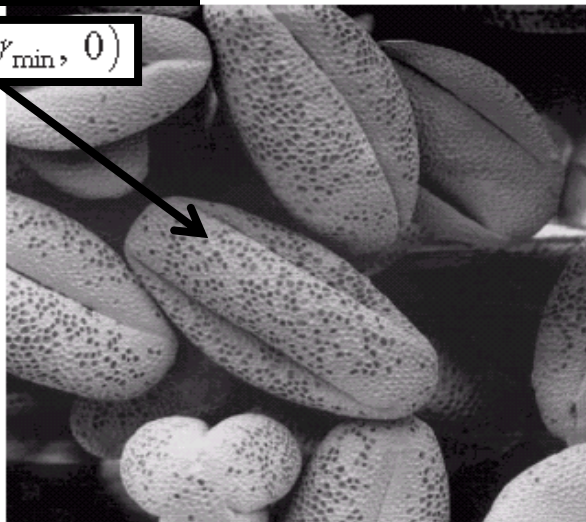
$$(r_1, s_1) = (r_{\min}, 0)$$



a b  
c d

**FIGURE 3.10**

Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



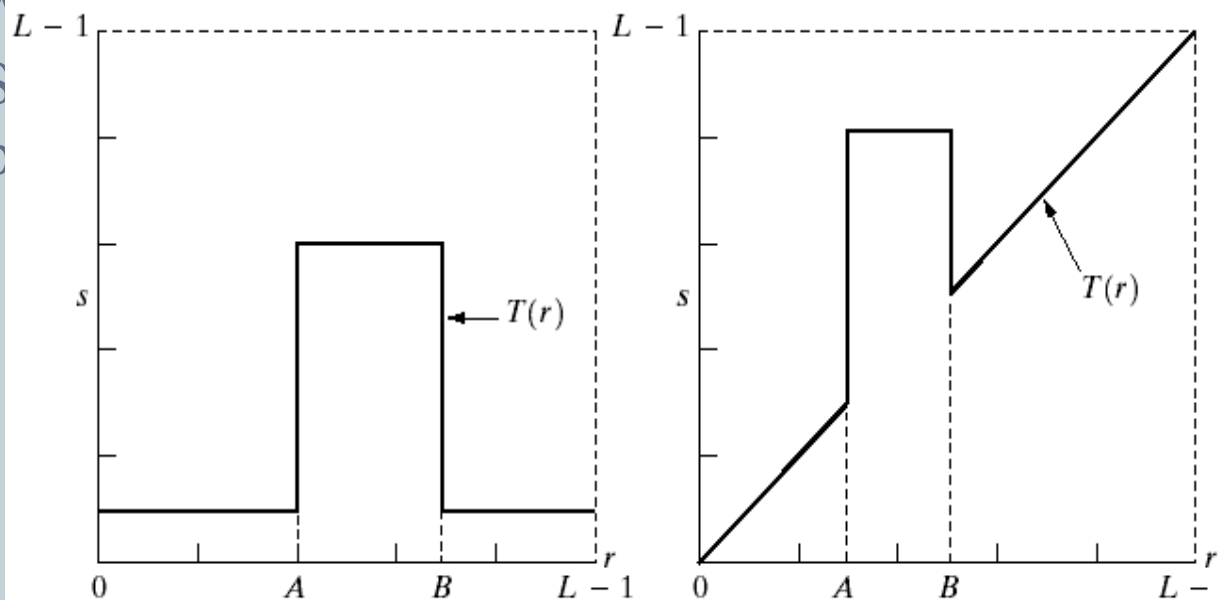
$$r_1 = r_2 = m_2$$

# Piecewise-Linear Transformation Functions (Cont...)

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- **Intensity-level slicing**

- Highlighting a specific range of intensities in an image often is desired.
- One approach is to highlight the range of intensities (say white) and reduce all other intensities to lower level.
- Another approach is to highlight the range of intensities (say black) and reduce all other intensities to a higher level.



a	b
c	d

**FIGURE 3.11**

(a) This transformation highlights range  $[A, B]$  of gray levels and reduces all others to a constant level. (b) This transformation highlights range  $[A, B]$  but preserves all other levels.

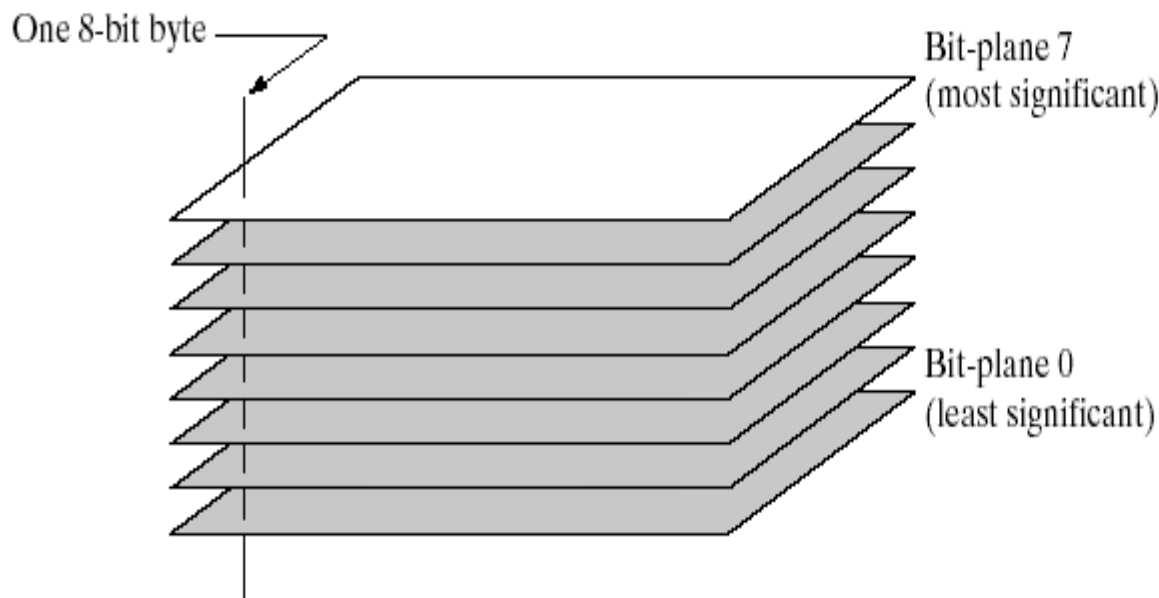


# Piecewise-Linear Transformation Functions (Cont...)

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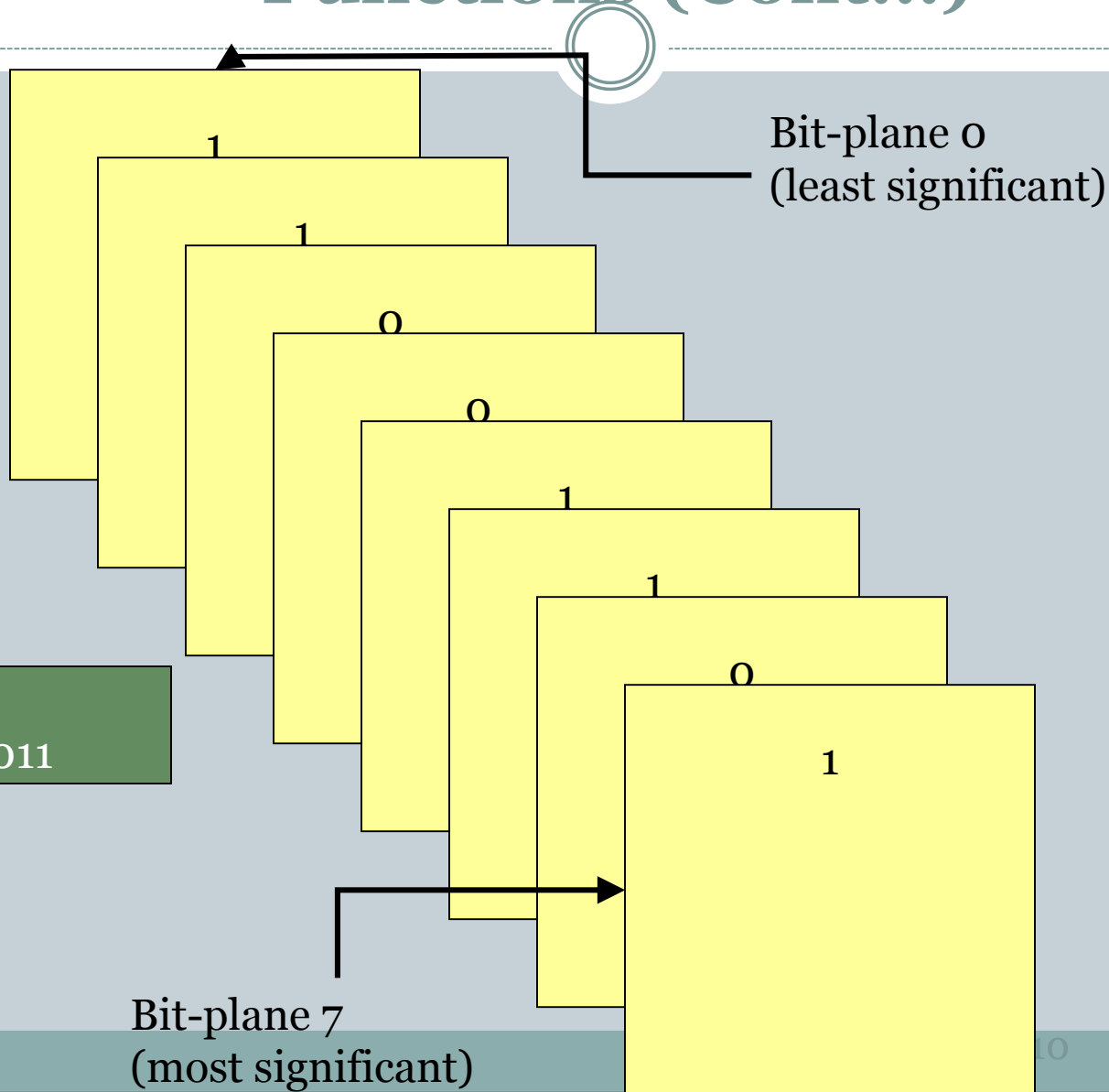
- **Bit-plane slicing**

- Often by isolating particular bits of the pixel values in an image we can highlight interesting aspects of that image.
- Higher-order bits usually contain most of the significant visual information.



**FIGURE 3.12**  
Bit-plane  
representation of  
an 8-bit image.

# Piecewise-Linear Transformation Functions (Cont...)



# Piecewise-Linear Transformation Functions (Cont...)

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- **Bit-plane slicing**



Bit-plane 7



Bit-plane 6



Bit-plane 4



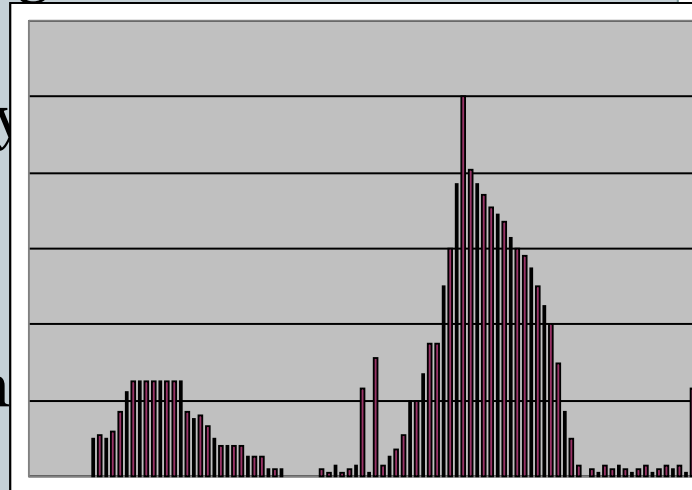
Bit-plane 1

# Histogram Processing

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- Histogram: A graph indicating the number of times each gray level occurs in the image and shows us the distribution of grey levels in the image.
- Grey level:  $0 \sim L-1$
- $L$  is the number of possible intensity
- Histogram function:  $h(r_k) = n_k$ 
  - $r_k$ : the  $k$ th grey level
  - $n_k$ : the # of pixel with grey level  $k$
- Normalized Histogram:  $P(r_k) = n_k / mn$ 
  - $mn$ : the # of total pixel
- Information inherent in histograms is quite useful in other image processing applications, such as image compression and segmentation.

Frequencies



Grey Levels

# Histogram Processing (Cont...)

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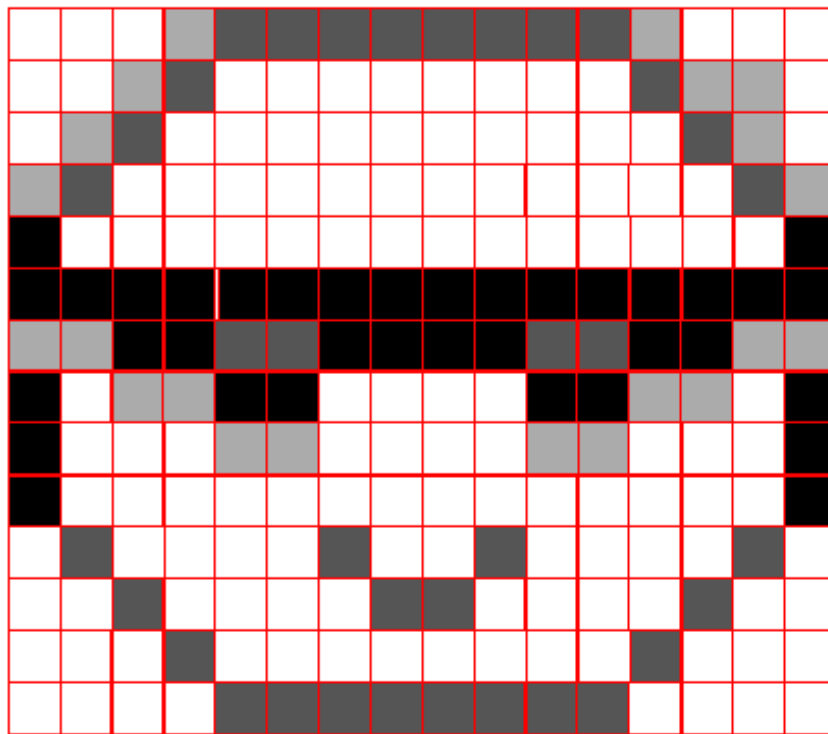
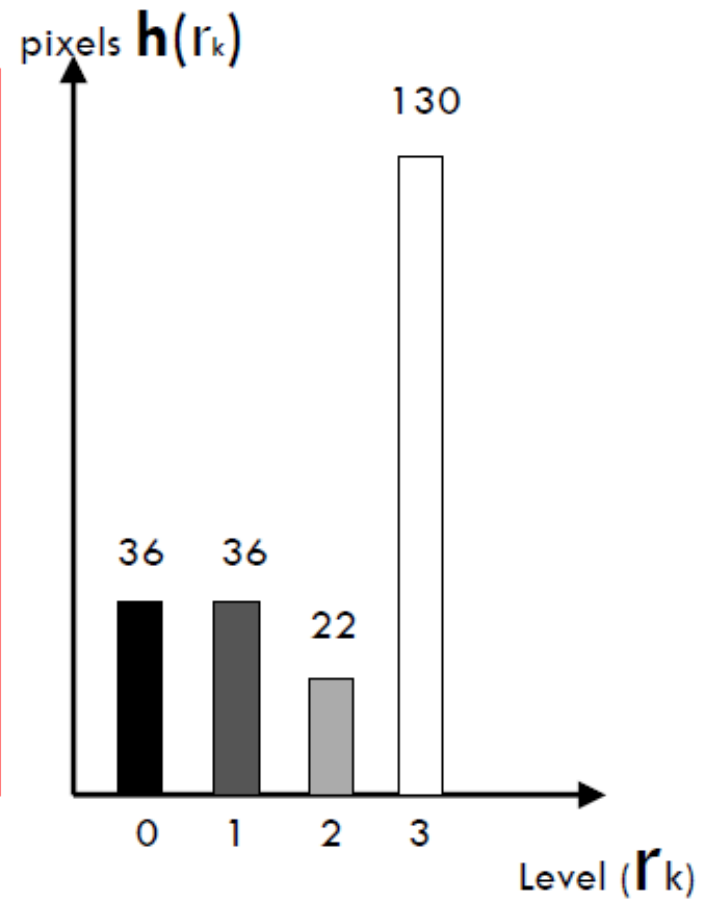


Image 16x14 = 224 pixels



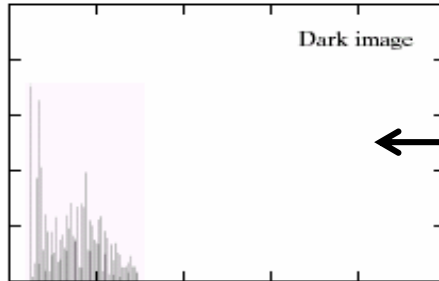
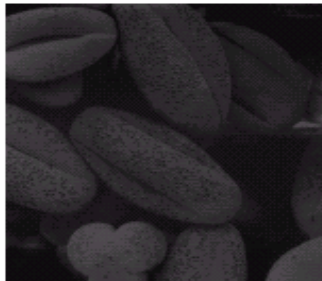
# Histogram Processing (Cont...)

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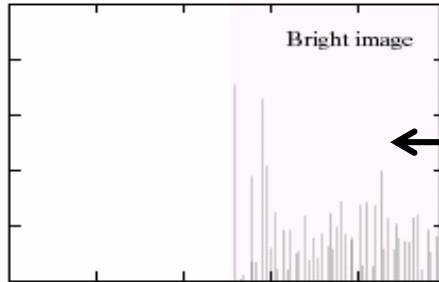
- The role of histogram processing in image enhancement example:
  - Dark image
  - Light image
  - Low contrast
  - High contrast

# Histogram Processing (Cont...)

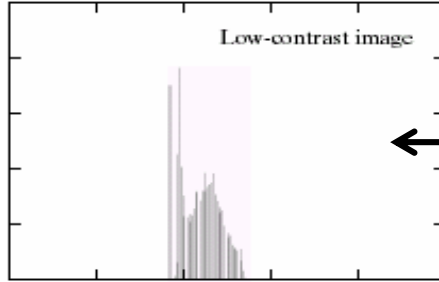
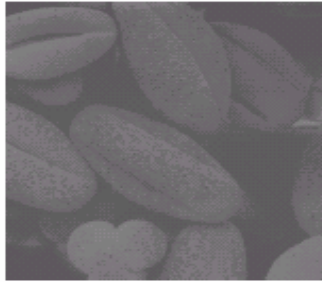
15



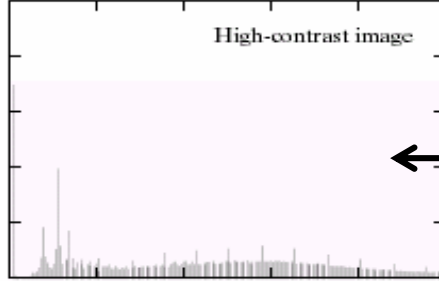
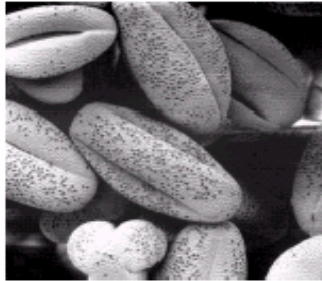
**histogram are concentrated on the low (dark) side of the gray scale.**



**histogram is biased toward the high side of the gray scale.**



**histogram will be narrow and will be centered toward the middle of the gray scale.**



**cover a broad range of the gray scale, with very few vertical lines being much higher than the others**

# Histogram Processing (Cont...)

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- Histogram Equalization
- Histogram Matching(Specification)
- Local Enhancement



# Histogram Equalization

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- Histogram equalization is a technique for adjusting image intensities to enhance contrast.
- $P_r(r_k)$ : The probability of occurrence of gray level  $r_k$ , where:

$$p_r(r_k) = \frac{n_k}{MN} \quad k = 0, 1, 2, \dots, L - 1$$

- This equation is histogram equalization or linearization

$$s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j)$$
$$= \frac{(L - 1)}{MN} \sum_{j=0}^k n_j \quad k = 0, 1, 2, \dots, L - 1$$

- Thus, a processed (output) image is obtained by mapping each pixel with level  $r_k$  in the input image into a corresponding pixel with level  $s_k$  in the output image.
  - $n_k$  : number of pixels that have gray level  $k$  ,
  - $mn$  : total number of pixels in the image,

# Histogram Equalization (Cont...)

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Suppose that a 3-bit image ( $L = 8$ ) of size  $64 \times 64$  pixels ( $MN = 4096$ ) has the intensity distribution shown in Table 3.1, where the intensity levels are integers in the range  $[0, L - 1] = [0, 7]$ .

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7p_r(r_0) = 1.33 \quad \left| \quad s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and  $s_2 = 4.55$ ,  $s_3 = 5.67$ ,  $s_4 = 6.23$ ,  $s_5 = 6.65$ ,  $s_6 = 6.86$ ,  $s_7 = 7.00$

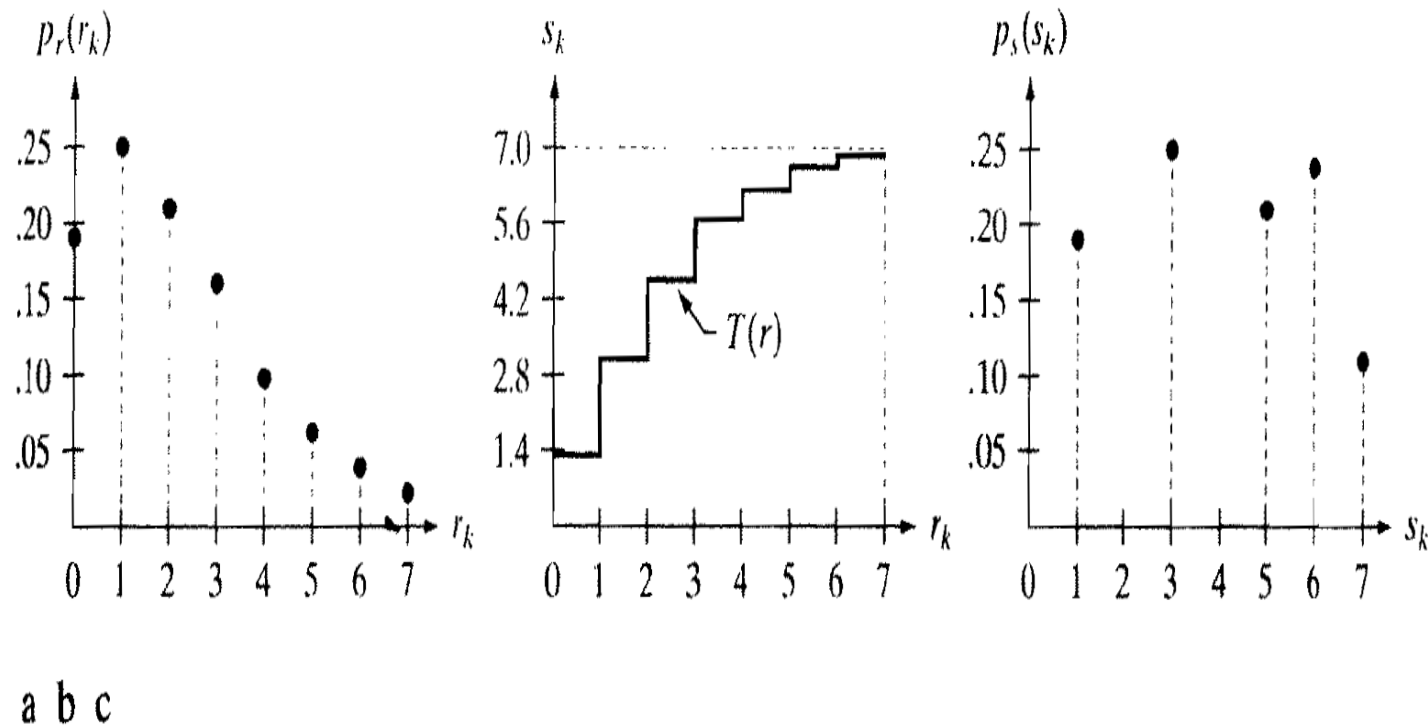
**TABLE 3.1**

Intensity distribution and histogram values for a 3-bit,  $64 \times 64$  digital image.

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

# Histogram Equalization (Cont...)

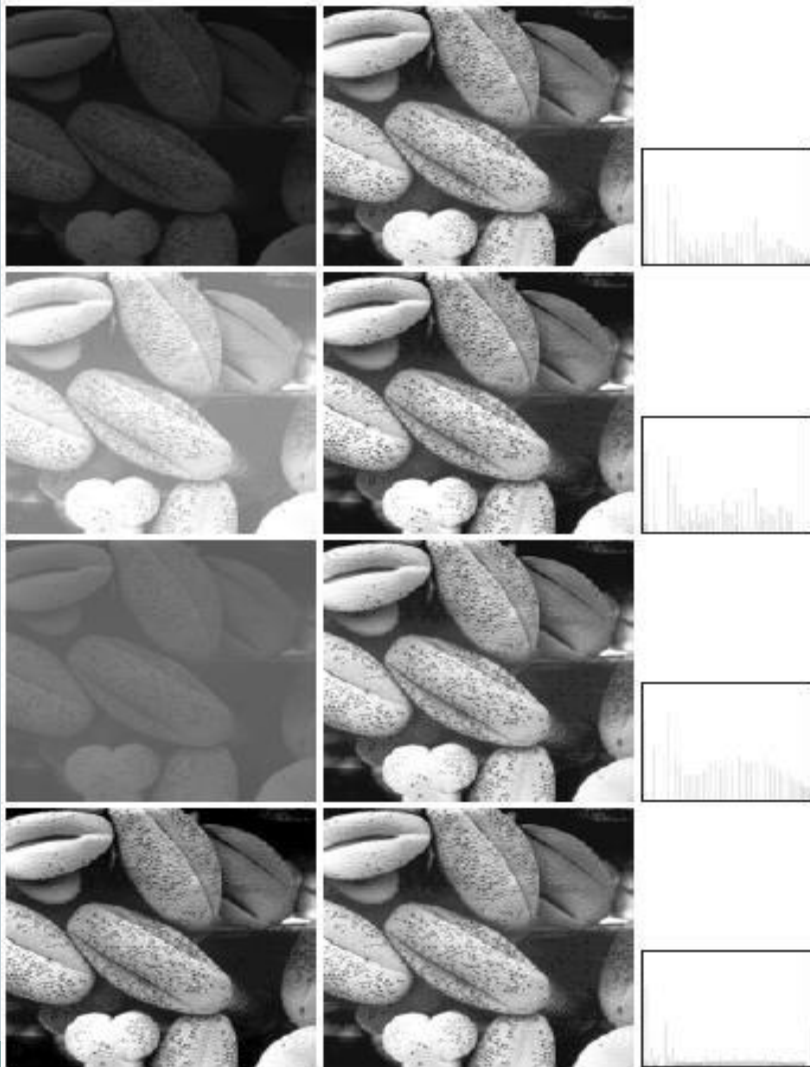
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**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

# Histogram Equalization (Cont...)

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- The first three results (top to bottom) show significant improvement.
- Histogram equalization did not produce a significant visual difference in the fourth image because the histogram of this image already spans the full spectrum of the gray scale.

# Histogram (Specification) Matching

- Histogram equalization automatically determines a transformation function that seeks to produce an output image that has a uniform histogram.
- Histogram matching is the method used to generate a processed image that has a specified histogram.
- In this notation,  $r$  and  $z$  denote the gray levels of the input and output (processed) images, respectively.

# Histogram (Specification) Matching (Cont...)

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- We can estimate  $p_r(r)$  from the given image while  $p_z(z)$  is the specified probability density function that we wish the output image to have.

- Let  $s$  be a random variable with the property

$$s = T(r) = \int_0^r p_r(w)dw \quad \dots(1)$$

- Suppose that we define a random variable  $z$  with property

$$G(z) = \int_0^z p_z(t)dt = s \quad \dots(2)$$

- Then two equations imply  $G(z)=T(r)$   
 $z = G^{-1}(s) = G^{-1}[T(r)] \quad \dots(3)$

# Histogram (Specification) Matching (Cont...)

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- Discrete formulation of Eq.(1) :

$$\begin{aligned} s_k &= T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j) \\ &= \frac{(L - 1)}{MN} \sum_{j=0}^k n_j \quad k = 0, 1, 2, \dots, L - 1 \end{aligned}$$

- Discrete formulation of Eq.(2) :

$$G(z_q) = (L - 1) \sum_{i=0}^q p_z(z_i)$$

- Discrete formulation of Eq.(3)

$$z_q = G^{-1}(s_k)$$

# Histogram (Specification) Matching (Cont...)

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## Procedures:

- Obtain the histogram of the given image.
- Use Eq.(1) to obtain the transformation function  $T(r)$ .
- Use Eq.(2) to obtain the transformation function  $G(z)$ .
- Obtain the inverse transformation function  $G^{-1}$ .
- Obtain the output image by applying Eq.(3) to all the pixels in the input image.



# Histogram (Specification) Matching (Cont...)

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$z_q$	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00$$

Similarly,

$$G(z_1) = 7 \sum_{j=0}^1 p_z(z_j) = 7[p(z_0) + p(z_1)] = 0.00$$

and

$$G(z_2) = 0.00 \quad G(z_4) = 2.45 \quad G(z_6) = 5.95$$

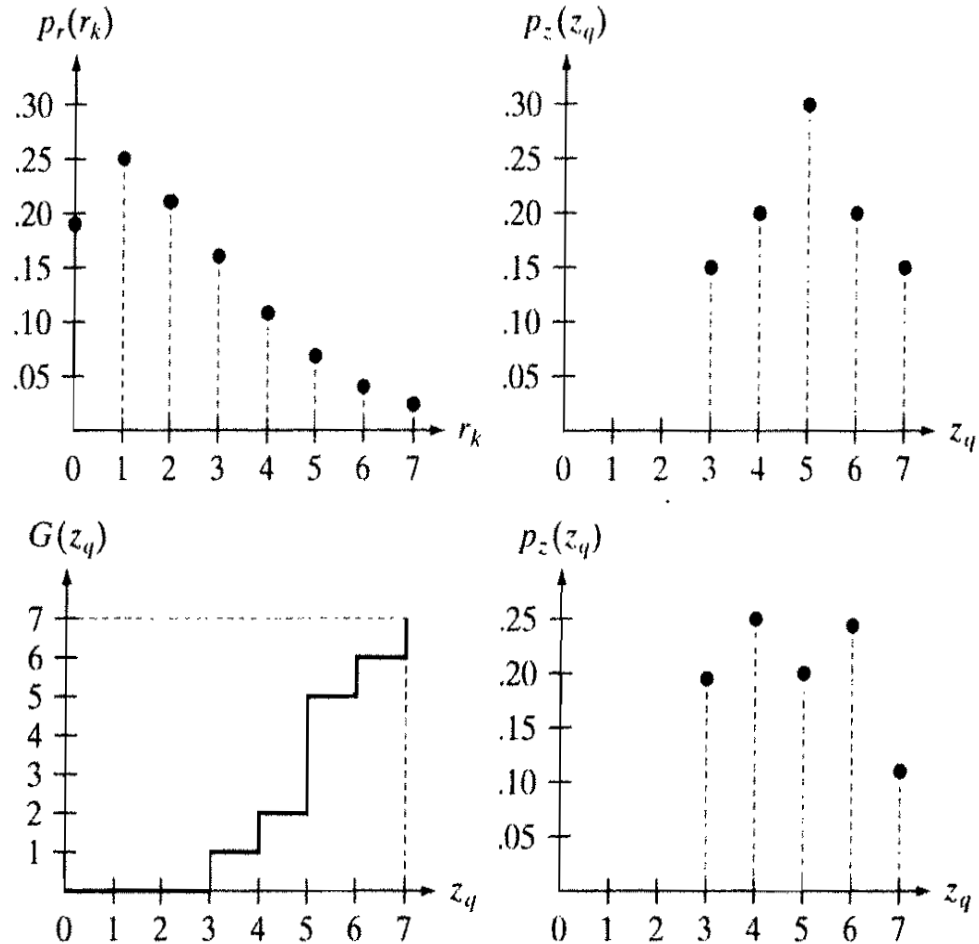
$$G(z_3) = 1.05 \quad G(z_5) = 4.55 \quad G(z_7) = 7.00$$

# Histogram (Specification) Matching (Cont...)

a b  
c d

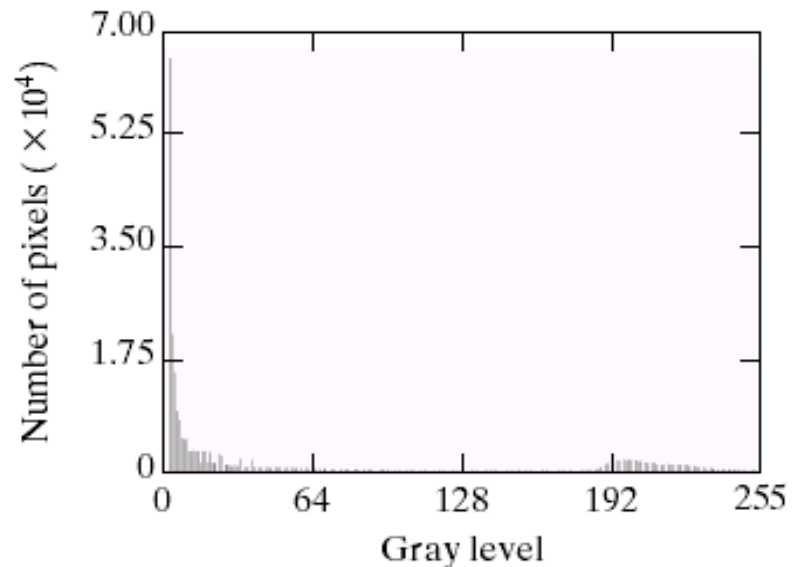
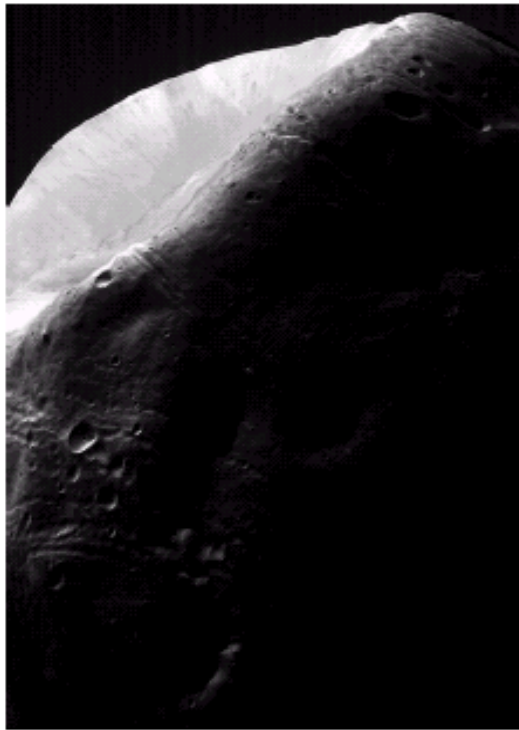
**FIGURE 3.22**

(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).



# Histogram (Specification) Matching (Cont...)

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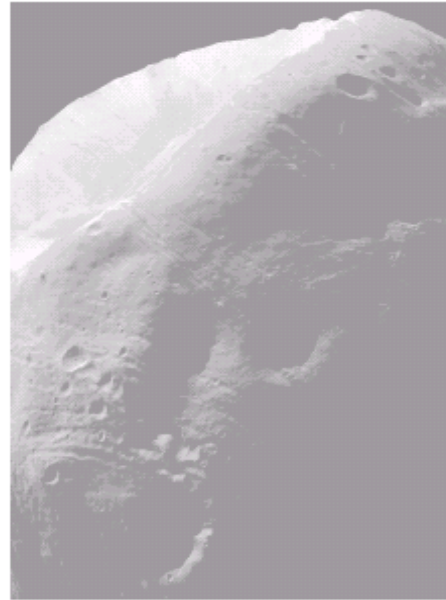
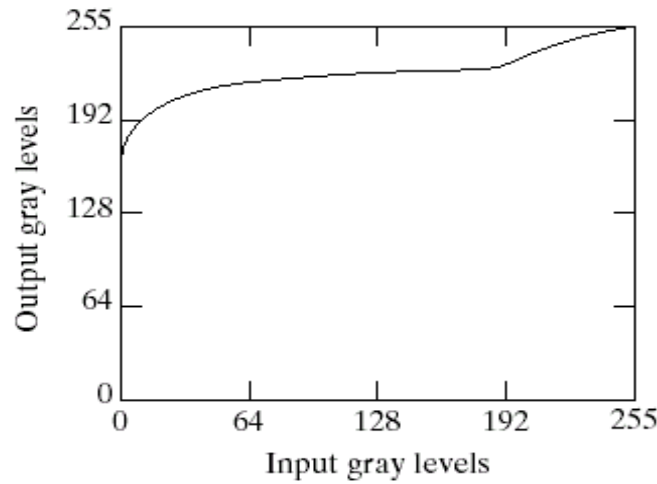


a b

**FIGURE 3.20** (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

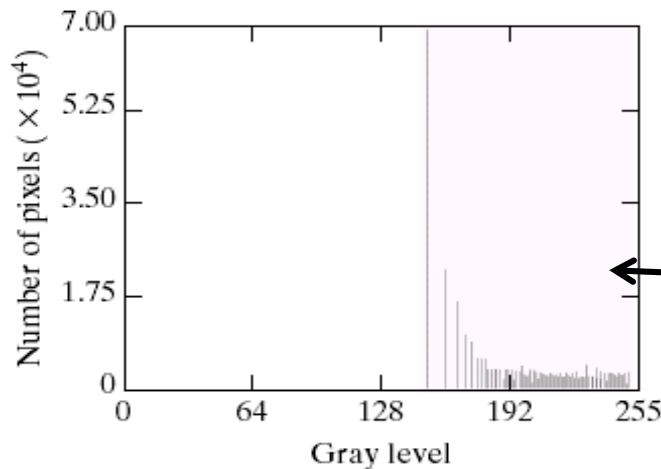
# Histogram (Specification) Matching (Cont...)

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a b  
c

**FIGURE 3.21**  
(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).



gray levels are biased toward the upper one-half of the gray scale.

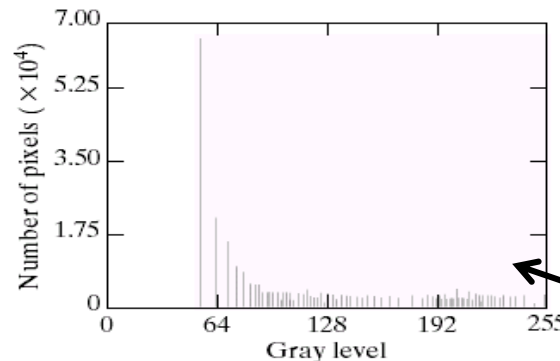
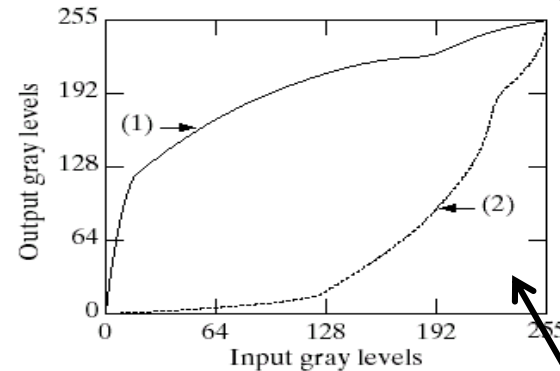
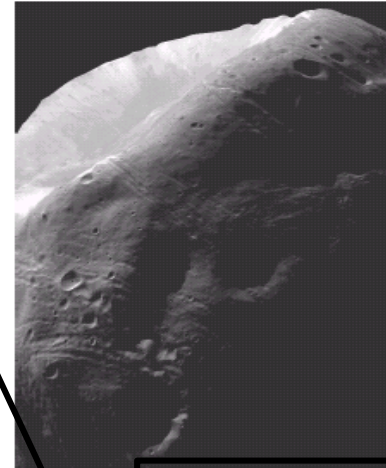
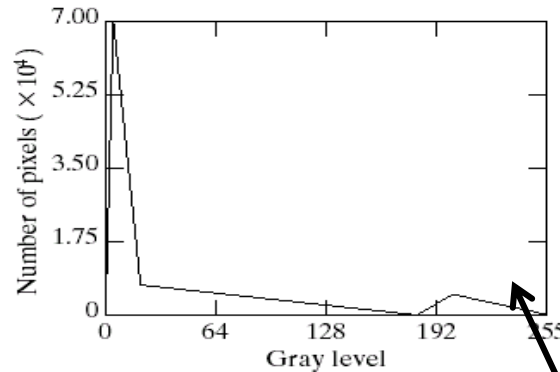
# Histogram (Specification) Matching (Cont...)

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a c  
b  
d

**FIGURE 3.22**

(a) Specified histogram.  
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).  
(c) Enhanced image using mappings from curve (2).  
(d) Histogram of (c).



***Specified function that preserves the general shape of the original histogram, but has a smoother transition of levels in the dark region of the gray scale.***

**From Eq(2) using histogram in (a)**

**Histogram of above**

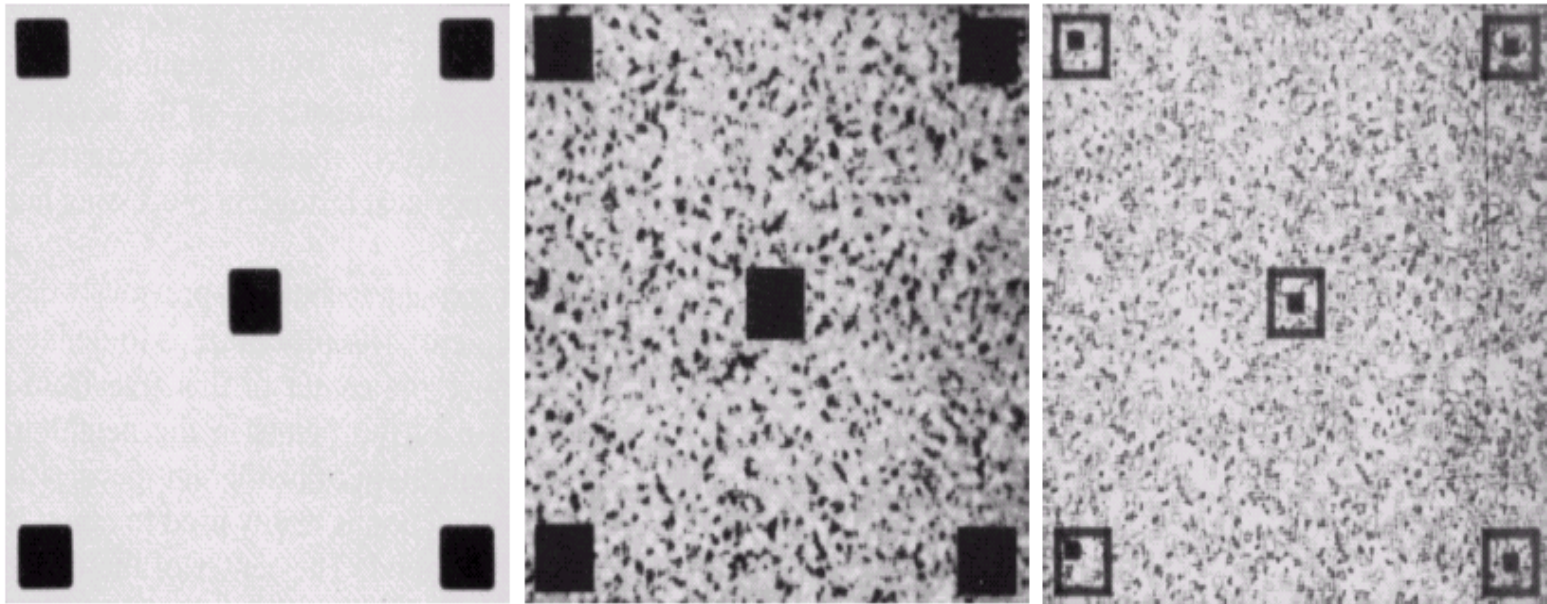
# Local Histogram Processing

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- There are cases in which it is necessary to enhance details over small areas in the image.
- The numbers of areas may have negligible influence on the computation of global transformation.
- The solution is to advise transformation function based on the intensity distribution in a neighborhood of every pixel in the image.
- The procedure is to define a neighborhood and move its center from pixel to pixel.

# Local Histogram Processing (Cont...)

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a b c

**FIGURE 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a  $7 \times 7$  neighborhood about each pixel.



# Using Histogram Statistics for Image Enhancement

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- The mean (Average intensity) of  $r$ :

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

- The intensity variance:

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

- The mean is a measure of average intensity and the variance is a measure of contrast in the image once the image histogram has been obtained.

- $m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$  sample variance:

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$



# Using Histogram Statistics for Image Enhancement (Cont...)

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- Let  $f(x, y)$  represent the value of an image pixel at any image coordinates  $(x, y)$ .
- let  $g(x, y)$  represent the corresponding enhanced pixel value.

$$g(x, y) = \begin{cases} E \cdot f(x, y) & \text{if } m_{S_{xy}} \leq k_0 M_G \quad \text{AND} \quad k_1 D_G \leq \sigma_{S_{xy}} \leq k_2 D_G \\ f(x, y) & \text{otherwise} \end{cases}$$

- Where:
  - $E, k_0, k_1, \text{ and } k_2$  are specified parameters.
  - $M_G$  is the global mean of the input image; and
  - $D_G$  is its global standard deviation.

# The Mechanics of Spatial Filtering

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- The spatial filter consists of:
  - Neighborhood, (typically a small rectangle).
  - Predefined operation that is performed on the image pixels by the neighborhood.
  - Filtering creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood , and whose value is the result of filtering operation.
  - Spatial Filtering:
    - ✦ Linear spatial filtering.
    - ✦ Nonlinear spatial filtering.

# The Mechanics of Spatial Filtering (Cont...)

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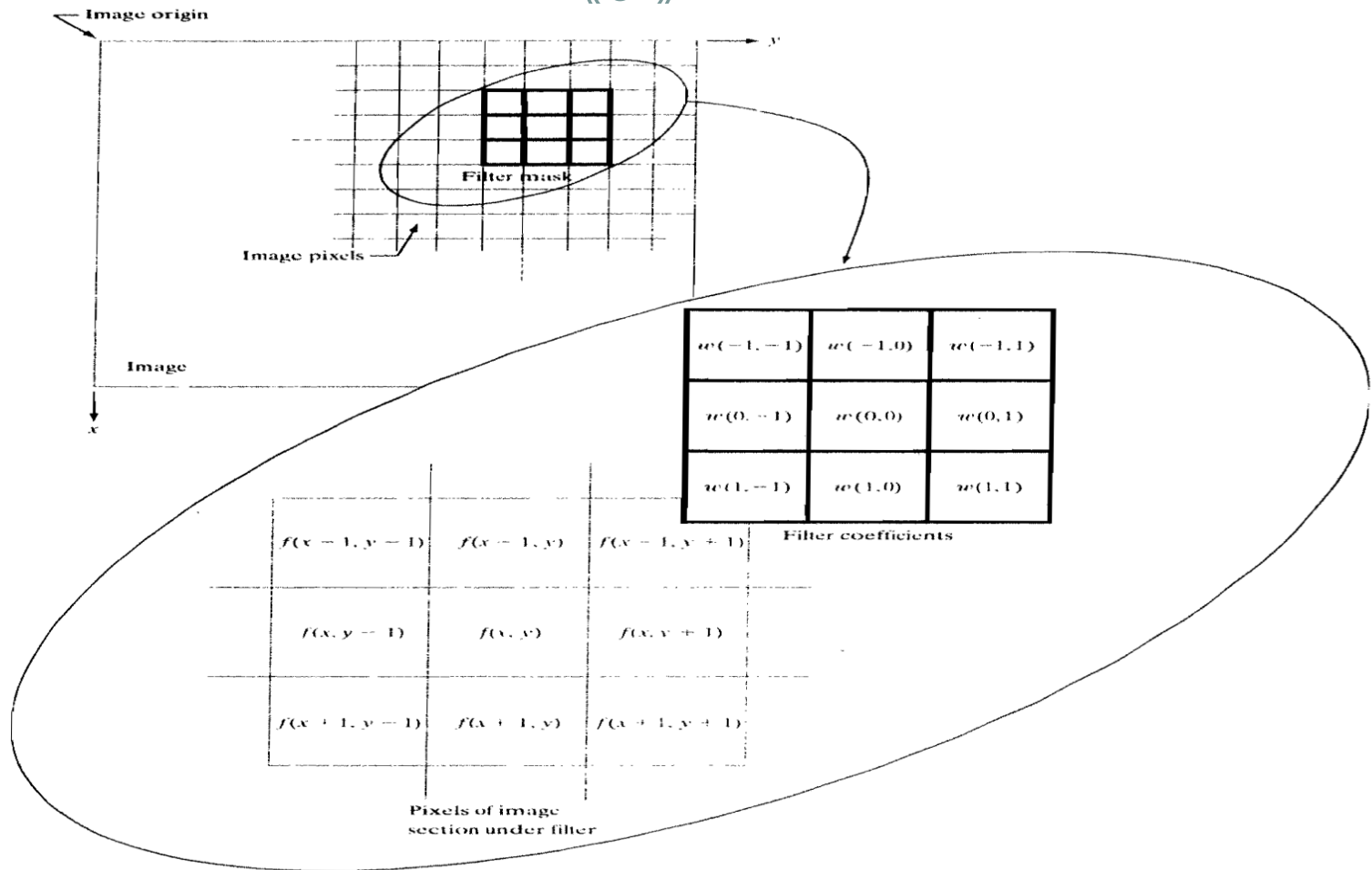
- The linear spatial filter for image of M x N:

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

- Where x and y are varied so that each pixel in w visits every pixels in f,  $a = (m-1)/2$  and  $b = (n-1)/2$ .
- The spatial filter should be odd size and the smallest being of size 3 x 3.

# The Mechanics of Spatial Filtering (Cont...)

(36)

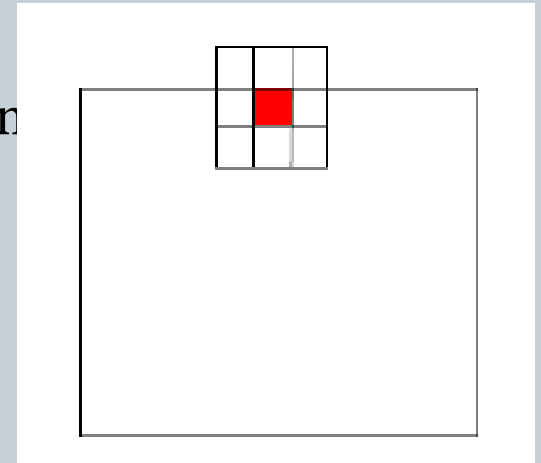


**FIGURE 3.28** The mechanics of linear spatial filtering using a  $3 \times 3$  filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

# What Happens at the Borders?

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- The mask falls outside the edge!
- Solutions?
  - Ignore the edges
    - ✦ The resultant image is smaller than the original
  - Pad with zeros
    - ✦ Introducing unwanted artifacts



# Spatial Correlation and Convolution

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- Correlation is the process of moving a filter mask over the image and computing the sum of products at each

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

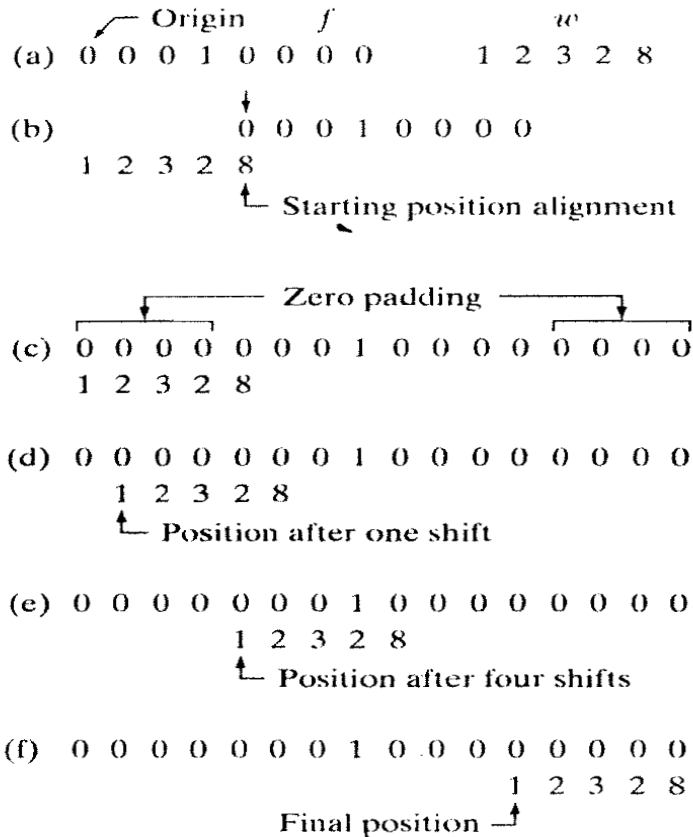
- The mechanics of convolution are the same except the filter

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

- Where the minus signs on the right flip  $f$  (i.e. rotate it by 180)

# Spatial Correlation and Convolution (Cont...)

## Correlation



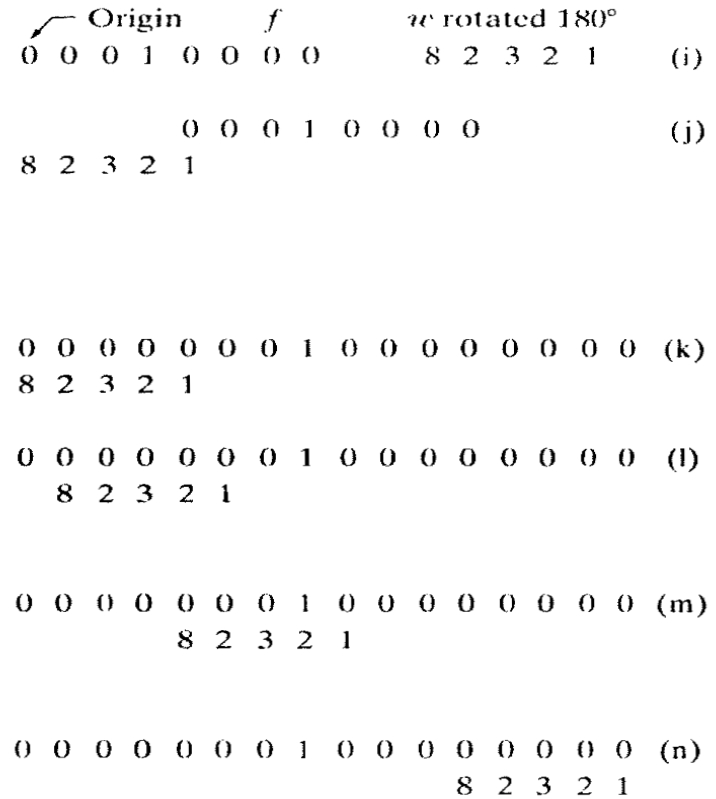
### Full correlation result

(g) 0 0 0 8 2 3 2 1 0 0 0 0

### Cropped correlation result

(h) 0 8 2 3 2 1 0 0

## Convolution



### Full convolution result

(o) 0 0 0 1 2 3 2 8 0 0 0 0

### Cropped convolution result

(p) 0 1 2 3 2 8 0 0

# Vector Representation of Linear Filtering

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- When interest lies in the characteristics response  $R$  of a mask either for correlation or convolution

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} \\ &= \sum_{k=1}^{mn} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$

- Where the  $w$ s are the coefficients of  $m \times n$  filter and  $z$ s are the corresponding image intensities encompassed by the filter.



# Generating Spatial Filter Masks

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- Generating an  $m \times n$  linear spatial filter requires that we specify  $m \times n$  mask coefficients.
- In turn, these coefficients are select based on what the filter supposed to do, keeping in your mind that all we can do with linear filtering is to implement a sum of products.

# Smoothing Spatial Filters

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- Smoothing filters are used for blurring and for noise reduction.
- Blurring is used in preprocessing tasks such as removal of small details from an image.
- Smoothing (i.e., low-pass filters)
  - Reduce noise and small details.
  - The elements of the mask must be positive.
  - Sum of mask elements is 1
- Noise reduction can be accomplished by:
  - Smoothing linear filter.
  - Smoothing nonlinear filter.

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

# Smoothing Linear Filters (Cont...)

43

- A **Low - Pass** filter allows low spatial frequencies to pass unchanged , but suppress high frequencies. The low pass filter smoothes or blurs the image.
- This tends to reduce noise, but also obscures fine detail.
- A spatial averaging filter in which all coefficients are equal sometimes is called a **box filter**.
- A spatial averaging filter in which the coefficients are different sometimes is called a **Weighted Average**.
- The following shows a 3 x 3 kernel for performing a low-pass filter operation. Each element in the kernel has a value of 1. The output pixel is just the simple average of the input neighborhood pixels.

$$h = 1/9 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$g(x, y) = \frac{1}{M} \sum_{j,k \in N} f(j, k)$$

# Smoothing Linear Filters (Cont...)



**FIGURE 3.33** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $m = 3, 5, 9, 15,$  and  $35$ , respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size  $50 \times 120$  pixels.

a b  
c d  
e f

# Order-Statistic (Nonlinear)Filters

45

- Order-statistic filters are nonlinear spatial filters whose response is based on ranking the pixels contained in the image area encompassed by the filter and then replacing the value of center pixel with the value determined by the ranking result.
- The best-known filter in this category is the median, min and max filters.
- Median filters are effective in the presence of impulse(salt and pepper) noise because of its appearance as white and black dots superimposed on an image.

# Order-Statistic (Nonlinear) Filters (Cont...)

46

- Very effective for removing “salt and pepper” noise (i.e., random occurrences of black and white pixels).

averaging

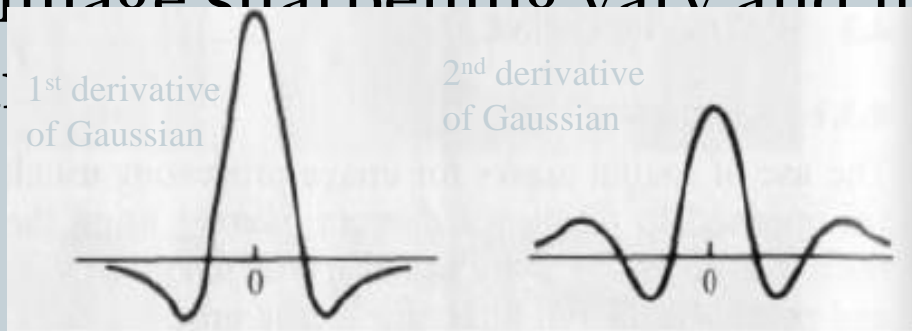
median  
filtering



# Sharpening Spatial Filters

47

- The principal objective of sharpening is to highlight transitions in intensity.
- Sharpening (i.e., high-pass filters)
  - Highlight fine detail or enhance detail that has been blurred.
  - The elements of the mask contain both positive and negative weights.
  - Sum of the mask weights is 0
- Uses of image sharpening vary and include applications in printing and medical imaging.



# Sharpening Spatial Filters (Cont...)

48

- Useful for emphasizing transitions in image intensity (e.g., edges).
- Sharpening filters are based on *spatial*

*di*

input image

10	10	10	10	10	10	80	80	80	
10	10	10	10	10	10	80	80	80	
10	10	10	10	10	10	80	80	80	
10	10	10	10	10	10	80	80	80	
10	10	10	10	10	10	80	80	80	

mask

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$\frac{1}{9} (-10 - 10 - 10 - 10 + 80 - 10 - 10 - 10 - 10) = 0$   
(there is no variation in the gray-levels)

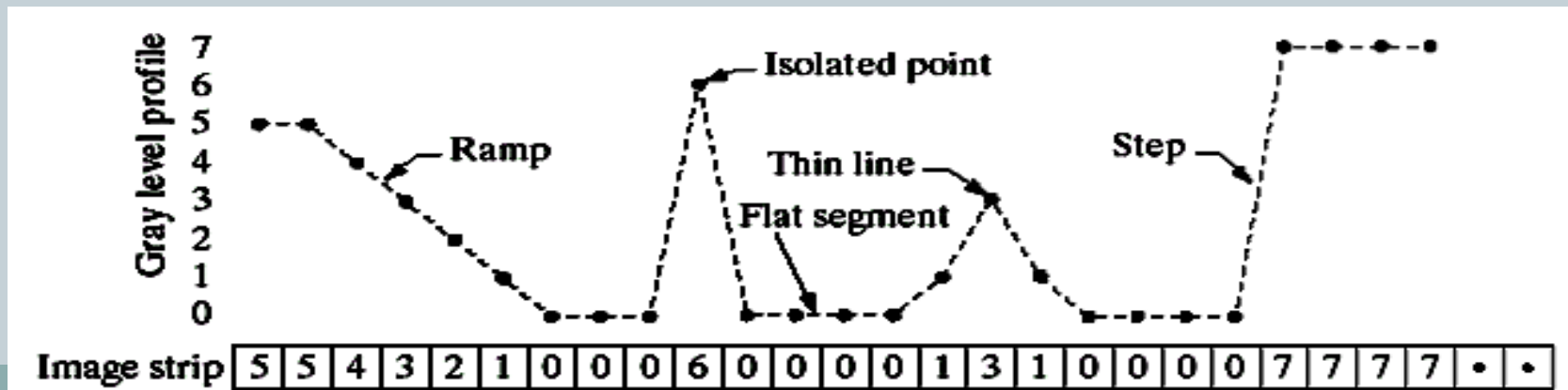
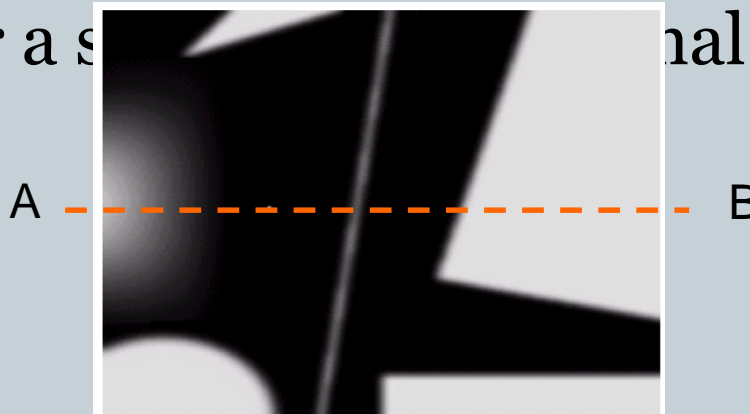
$\frac{1}{9} (-10 - 80 - 80 - 10 + 640 - 80 - 10 - 80 - 80) = \frac{210}{9} > 0$   
(there is variation in the gray-levels)



# Spatial Differentiation

49

- Differentiation measures the *rate of change* of a function.
- Let's consider a spatial example:



# 1<sup>st</sup> Derivative

50

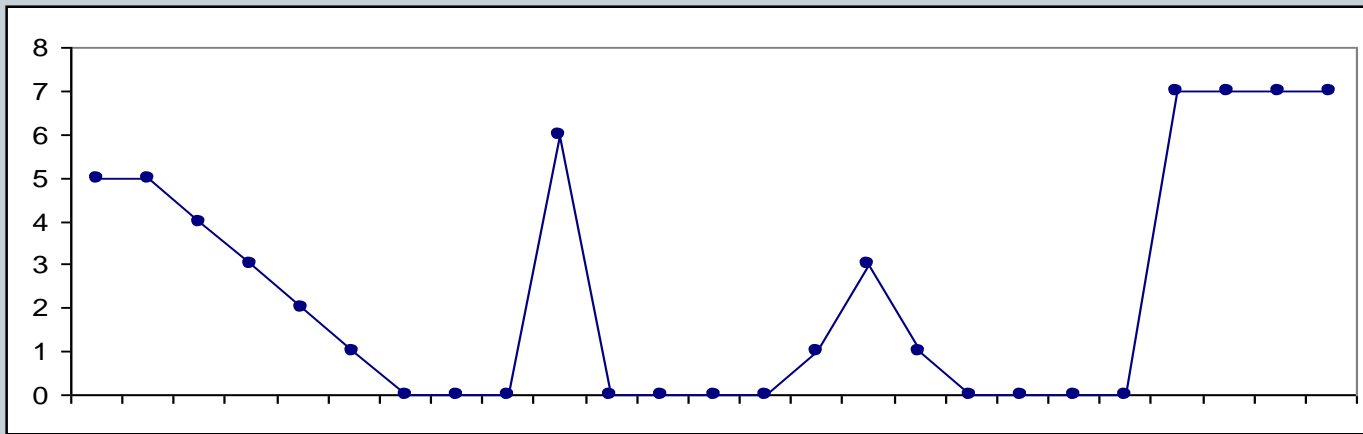
- The formula for the 1<sup>st</sup> derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x + 1) - f(x)$$

- It's just the difference between subsequent values and measures the rate of change of the function.

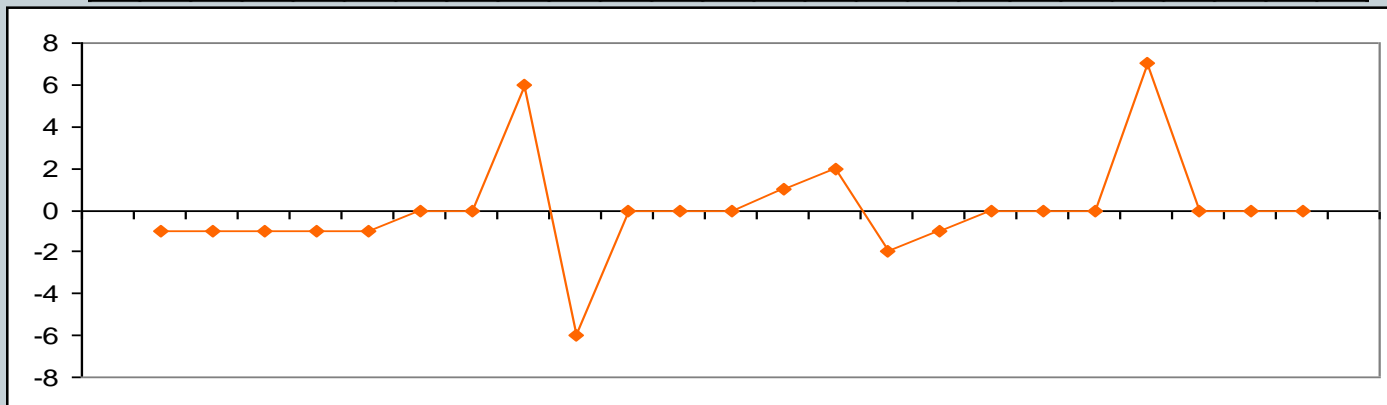
# 1<sup>st</sup> Derivative (Cont...)

51



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

0	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0
---	----	----	----	----	---	---	---	----	---	---	---	---	---	----	----	---	---	---	---	---	---	---



# 2<sup>nd</sup> Derivative

52

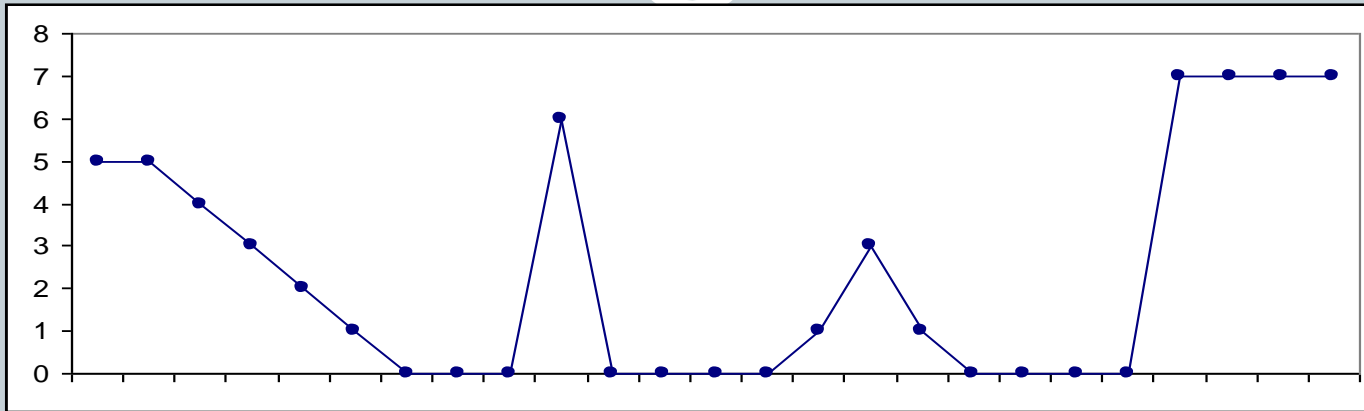
- The formula for the 2<sup>nd</sup> derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

- Simply takes into account the values both before and after the current value.

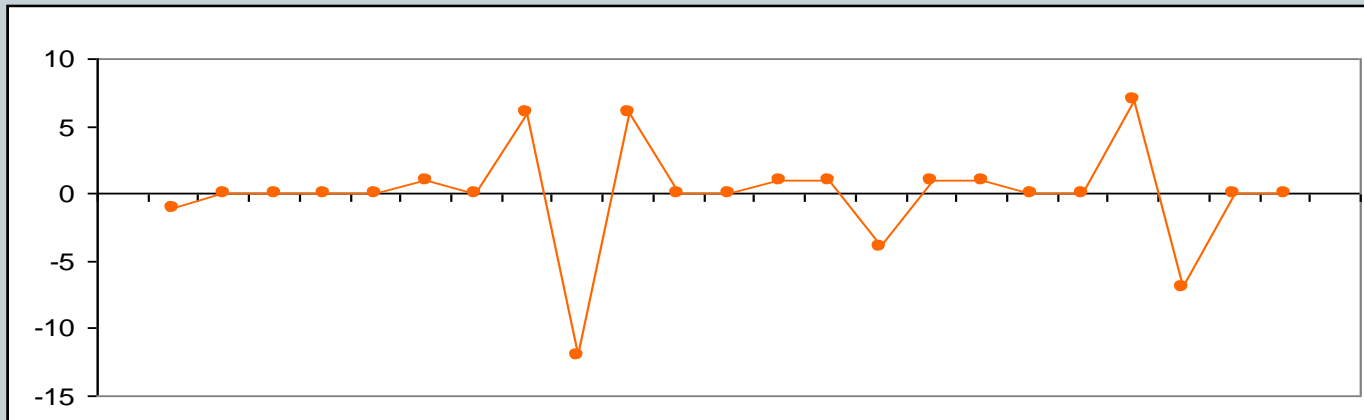
# 2<sup>nd</sup> Derivative (Cont...)

53



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0
----	---	---	---	---	---	---	---	-----	---	---	---	---	---	----	---	---	---	---	---	----	---	---



# Using Second Derivatives For Image Enhancement

54

- The 2<sup>nd</sup> derivative is more useful for image enhancement than the 1<sup>st</sup> derivative
  - Stronger response to fine detail
  - Simpler implementation
- The first sharpening filter we will look at is the *Laplacian*
  - One of the simplest sharpening filters
  - We will look at a digital implementation

# The Laplacian

55

- The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

- Where the partial 2<sup>nd</sup> order derivative in the  $x$  direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the  $y$  direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

# The Laplacian (Cont...)

56

- So, the Laplacian can be given as follows:

$$\begin{aligned}\nabla^2 f = & [f(x+1, y) + f(x-1, y) \\ & + f(x, y+1) + f(x, y-1)] \\ & - 4f(x, y)\end{aligned}$$

- We can easily build a filter based on this:

0	1	0
1	-4	1
0	1	0



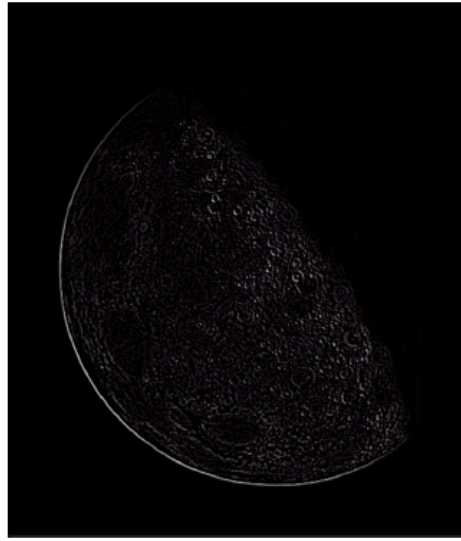
# The Laplacian (Cont...)

57

- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities



Original  
Image



Laplacian  
Filtered Image



Laplacian  
Filtered Image  
Scaled for Display

# The Laplacian (Cont...)

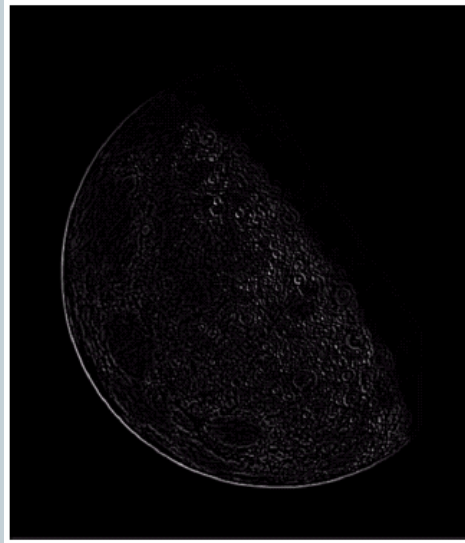
58

- The result of a Laplacian filtering is not an enhanced image so the final enhanced image:  
$$g(x, y) = f(x, y) - \nabla^2 f$$



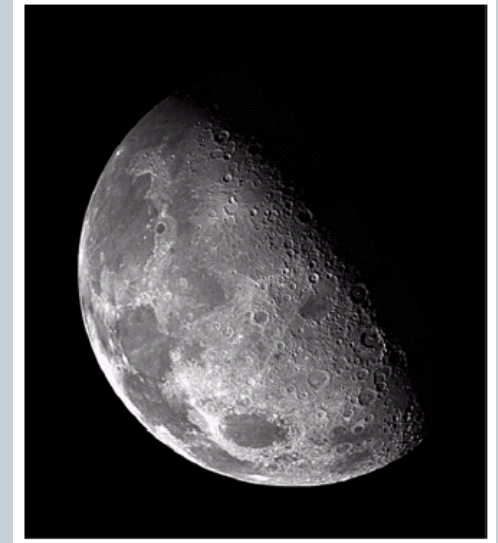
Original  
Image

-



Laplacian  
Filtered Image

=



Sharpened  
Image

# Variants On The Simple Laplacian

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- There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

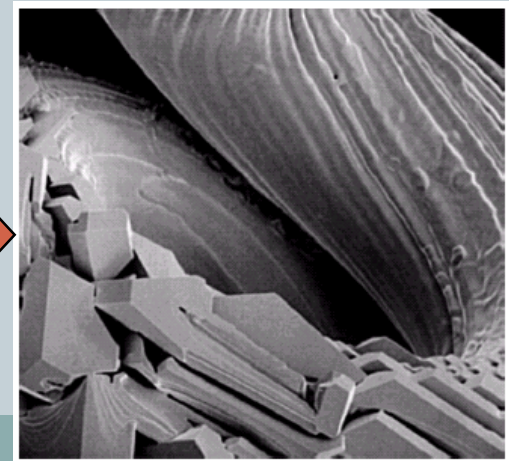
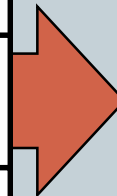
Simple  
Laplacian

1	1	1
1	-8	1
1	1	1

Variant of  
Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



# Sobel Operators

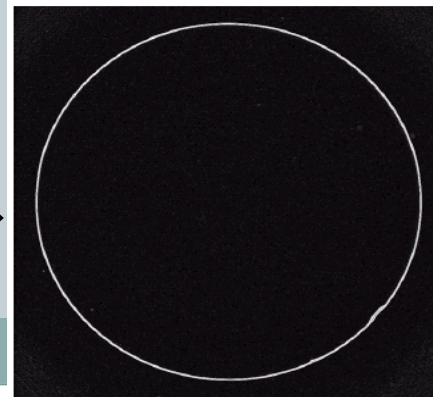
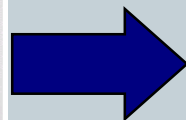
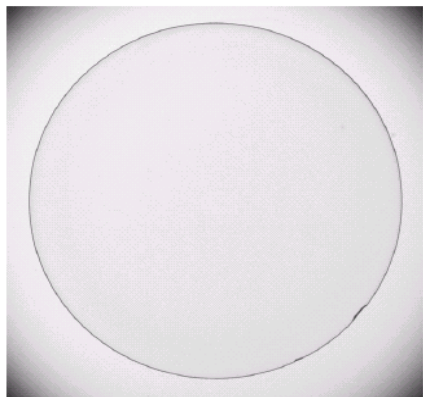
60

- To filter an image it is filtered using both operators the results of which are added together

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

- Sobel filters are typically used for edge detection.



An image of a contact lens which is enhanced in order to make defects (at four and five o'clock in the image) more obvious

# Unsharp Masking

61

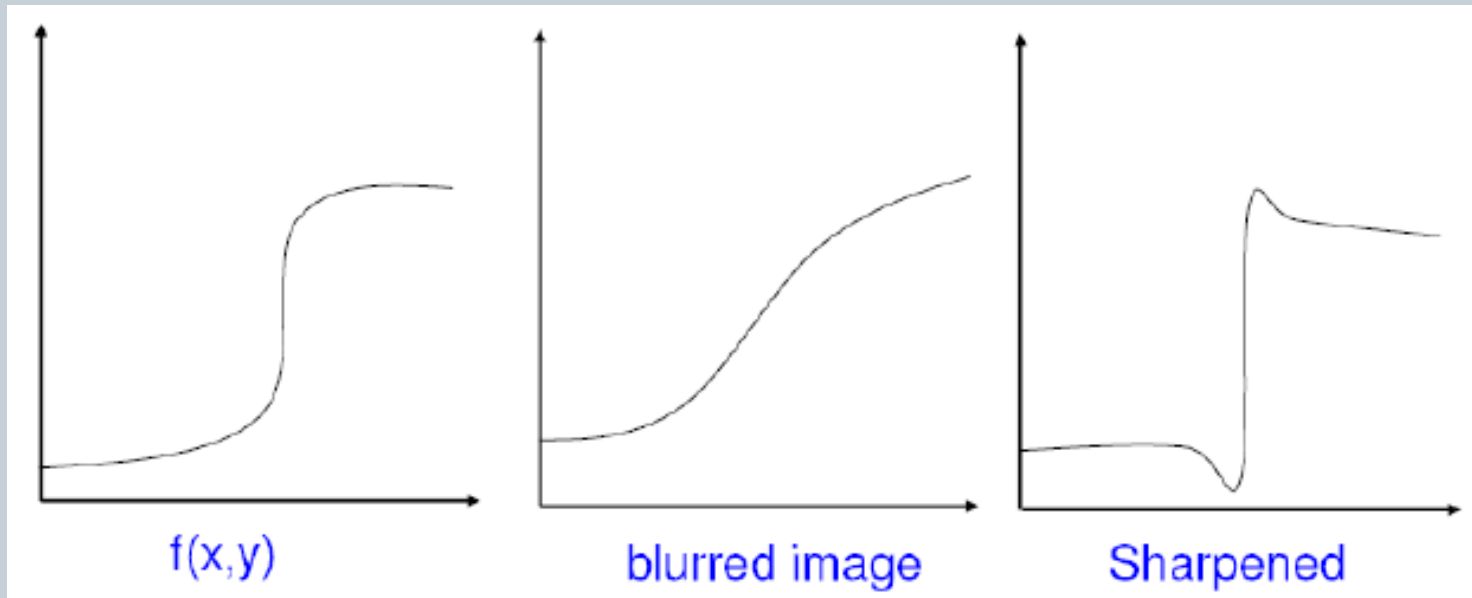
- A process used many years in the publishing industry to sharpen images
- Sharpened image: Subtracting a blurred version of an image from the image itself

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$

sharpened image = original image - blurred image

# Unsharp Masking (Cont...)

62



# Sharpening Filters: High Boost

63

- Image sharpening emphasizes edges but details (i.e., low frequency components) might be lost.
- **High boost filter:** amplify input image, then subtract a lowpass image

$$\text{Highboost} = A \text{ Original} - \text{Lowpass}$$

$$= (A - 1) \text{ Original} + \text{Original} - \text{Lowpass}$$

$$= (A - 1) \text{ Original} + \text{Highpass}$$

# Sharpening Filters: High Boost (Cont...)

64

- If  $A=1$ , we get a high pass filter.
- If  $A>1$ , part of the original image is added back to the high pass filtered image.

$$A \geq 1$$
$$w = 9A - 1$$

-1	-1	-1
-1	w	-1
-1	-1	-1

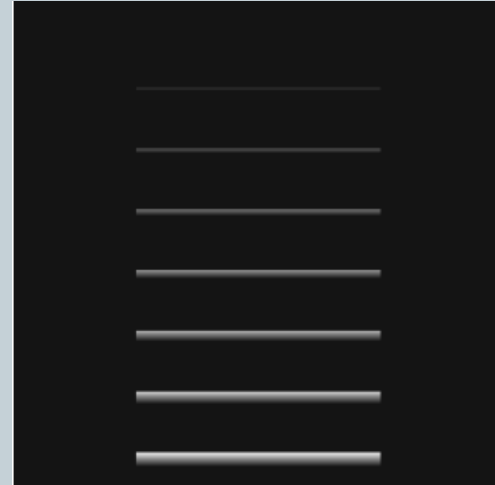
$$A = 2$$
$$w = 17$$

-1	-1	-1
-1	17	-1
-1	-1	-1

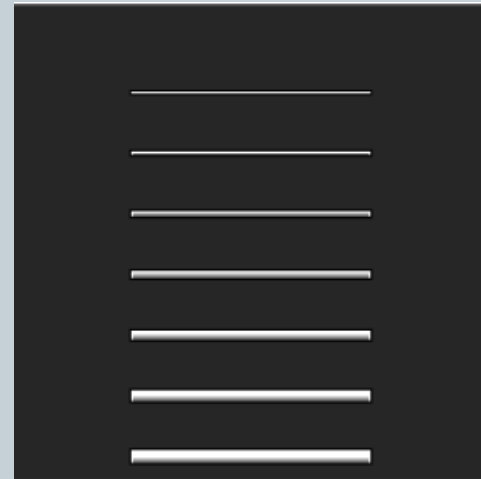


# Sharpening Filters: High Boost (Cont...)

65



A=1.4

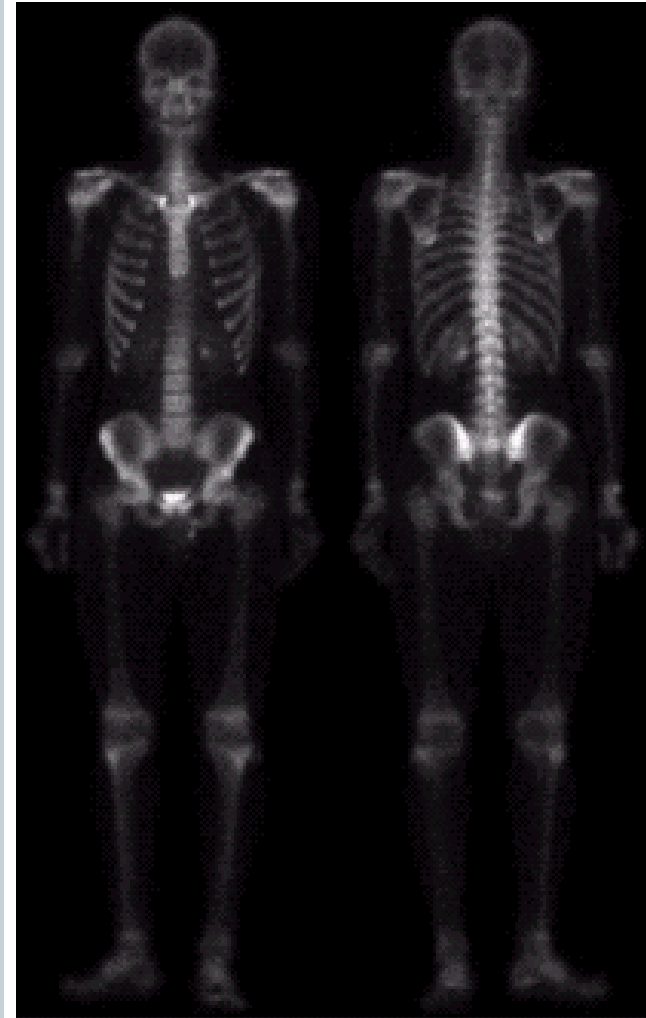


A=1.9

# Combining Spatial Enhancement Methods

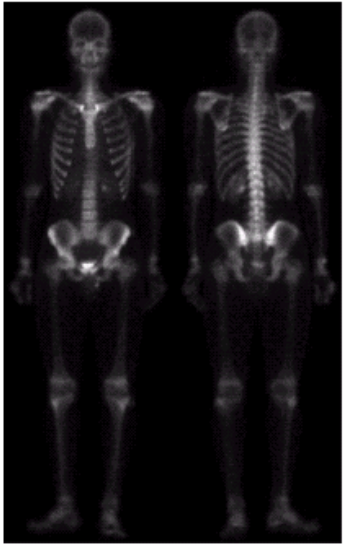
66

- Successful image enhancement is typically not achieved using a single operation.
- Rather we combine a range of techniques in order to achieve a final result.
- This example will focus on enhancing the bone scan to the right.



# Combining Spatial Enhancement Methods (Cont...)

67



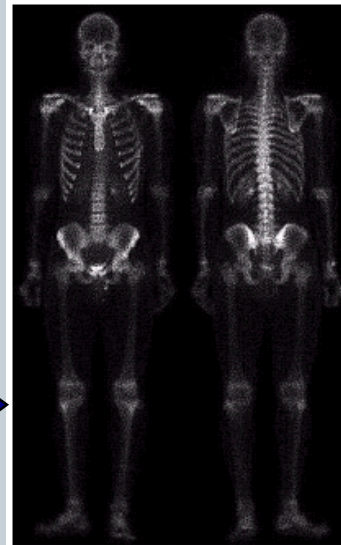
(a)

Laplacian filter of  
bone scan (a)



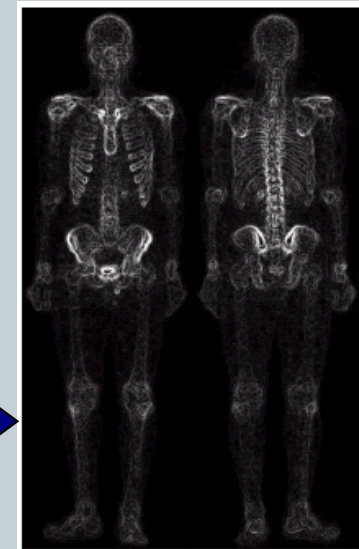
(b)

Sharpened version of  
bone scan achieved  
by subtracting (a)  
and (b)



(c)

Sobel filter of bone  
scan (a)

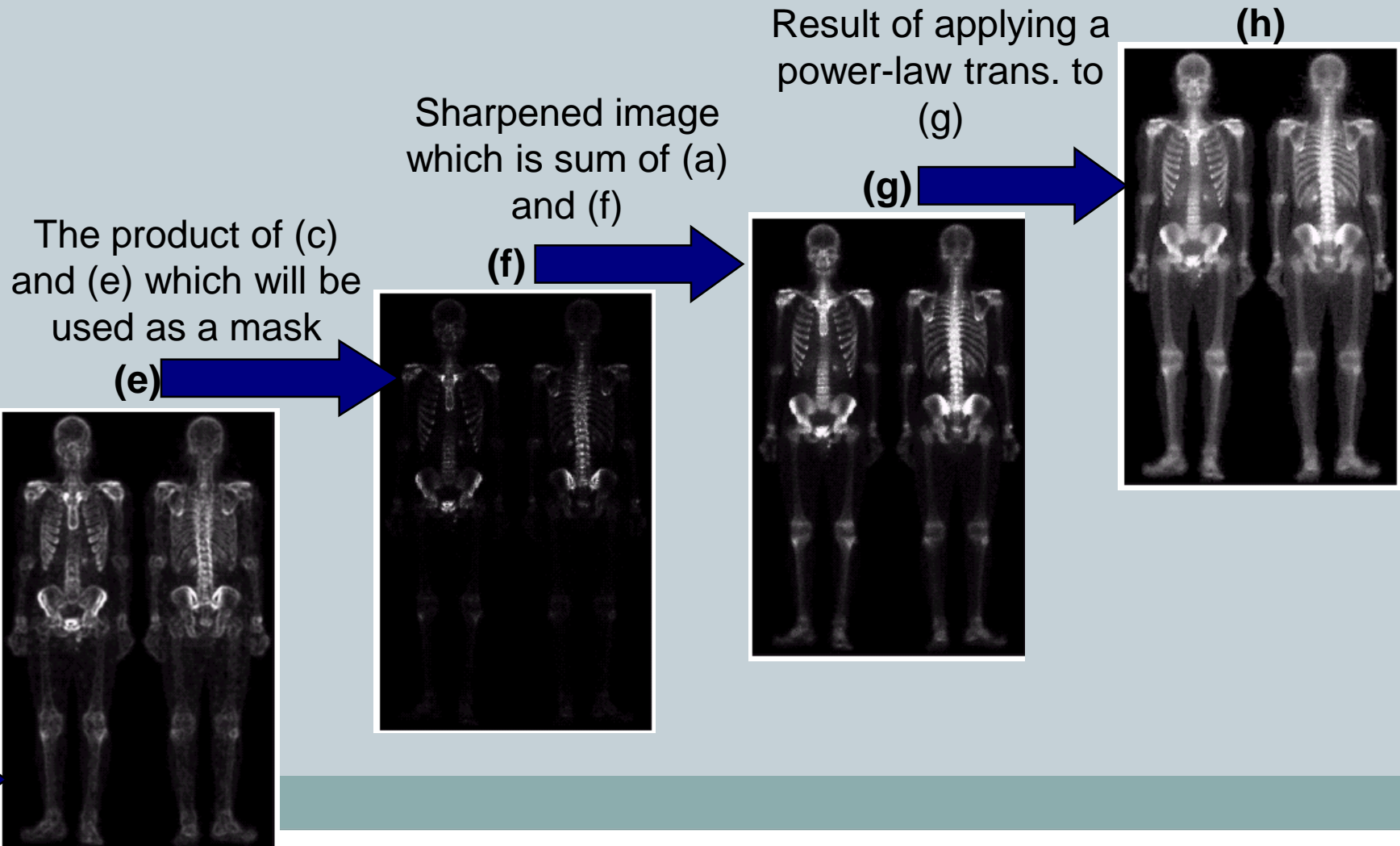


(d)

Image (d) smoothed  
with a 5\*5  
averaging filter

# Combining Spatial Enhancement Methods (Cont...)

68



# Combining Spatial Enhancement Methods (Cont...)

69

- Compare the original and final images:

